



Notes

QWERTY is efficient <sup>☆</sup>

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**Abstract**

We study a dynamic coordination problem with staggered decisions where agents choose between two competing networks. If the intrinsically worst one prevails, this is efficient. Moreover, inefficient shifts to the intrinsically best network might occur in equilibrium.

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**1. Introduction**

Consider the problem of choosing between two industry standards or networks (PC or Mac, iOS or Android, Facebook or Google+, DVD or blu-ray, QWERTY or Dvorak keyboards). A consumer takes into account not only the intrinsic quality of each alternative but also the number of people in each one. Agents' choices are strategic complements: the larger the amount of people in a given network, the more each individual is willing to choose that option. Moreover, these choices are only occasionally made, typically when our current device is old or not working very well. Hence, our decisions are staggered and expectations about the future are crucial.

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We observe that in many cases agents tend to follow the crowd and choose, say, Windows over Linux (even though many computer experts would recommend Linux), because it is useful to be in a large network of users.<sup>1</sup> Likewise, in the past century, we observed the mass adoption of the QWERTY keyboard over the Dvorak alternative because most people were used to the QWERTY standard, even though the Dvorak keyboard was arguably better in terms of its intrinsic quality.<sup>2</sup> This raises questions about efficiency in this problem. Is the equilibrium inefficient? Is there room for policy?

In order to answer these questions, we study efficiency in a dynamic coordination game with staggered decisions and show that the planner assigns an even lower weight to the intrinsic quality of each good than the agents. Hence the planner would be even more inclined towards QWERTY. One implication is that, if there is no other relevant externality, we should not subsidize a shift to an intrinsically better network – agents will move too early even without subsidies.

This paper builds on the model of Frankel and Pauzner (2000). They base their analysis on a model of sectorial choice (along the lines of Matsuyama, 1991), but their framework has been used to analyze location choices (Frankel and Pauzner, 2002), carry trades and speculation (Plantin and Shin, 2006), speculative attacks (Daniëls, 2009) and business cycles (Frankel and Burdzy, 2005; Guimaraes and Machado, 2015).<sup>3</sup> The model of currency attacks in Guimaraes (2006) and the model of debt runs in He and Xiong (2012) employ similar timing frictions.<sup>4</sup>

The paper is also related to the literature on network externalities, in which strategic complementarities arise from consumption externalities.<sup>5</sup> Agents' optimal choices typically depend on what they expect others will do. However, most of this literature makes ad-hoc assumptions on how agents coordinate.<sup>6</sup> Argenziano (2008), an important exception, considers a static coordination game played by ex ante heterogeneous agents who must choose among two networks. In her model, agents give too much importance to their own idiosyncratic tastes, so the efficient allocation would feature a larger higher-quality network. Here, in contrast, inefficiencies result from the dynamic interaction among agents: they sometimes switch to the higher-quality network when it would be socially better for them to stay in the lower-quality one.<sup>7</sup>

## 2. The model

There is a continuum of infinitely-lived agents indexed by  $i \in [0, 1]$ . Time is continuous and agents discount the future at rate  $\rho$ . An agent can be in two possible networks. We denote by  $a_{i,t} \in$

<sup>1</sup> In August 2015, about 85% of desktop or laptop computers worldwide used Microsoft Windows (Statcounter).

<sup>2</sup> See David (1985).

<sup>3</sup> In Guimaraes and Pereira (2015), we extend the model of Frankel and Pauzner (2000) to the case of ex-ante heterogeneous agents.

<sup>4</sup> This work is also related to the literature on coordination in games with strategic complementarities and asymmetric information, such as Carlsson and Van Damme (1993) and Morris and Shin (1998). The relation between this literature and that on dynamic coordination games (e.g., Frankel and Pauzner, 2000 and Burdzy et al., 2001) is discussed in Morris (2014).

<sup>5</sup> This literature has started with Katz and Shapiro (1985) and Katz and Shapiro (1986). See Shy (2011) for a survey.

<sup>6</sup> For instance, Katz and Shapiro (1986) assume that whenever there are multiple equilibria in the model, agents manage to coordinate their decisions in order to achieve the Pareto-superior outcome.

<sup>7</sup> In Argenziano's (2008), the higher-quality network is always the largest one in equilibrium. Hence her model is not well suited to analyze situations in which the largest observed network is the one with lowest intrinsic quality. Here, owing to the dynamic aspect of our model, the economy may be in states where the lower-quality network is the largest one in equilibrium – which captures well situations like the Linux–Windows dispute or the QWERTY–Dvorak choice.

$\{0, 1\}$  the network in which agent  $i$  is in at a given time  $t$ . Agents receive chances to revise their choice of network according to a Poisson process with arrival rate  $\delta$ , and stay committed to this network until the arrival of another opportunity. This timing friction might represent a machine break-up in an environment with a choice between two technologies, an attention friction of consumers or firms, or maturity of debt in a model of debt runs.<sup>8</sup>

The flow utilities agent  $i$  derives from being in networks 0 or 1 are given, respectively, by:

$$u_t^0(\theta_t^0, n_t) = \theta_t^0 + \nu(1 - n_t) \text{ and } u_t^1(\theta_t^1, n_t) = \theta_t^1 + \nu n_t,$$

where  $\theta_t^j$  represents the fundamentals affecting the flow-payoff of network  $j$  at time  $t$  for  $j \in \{0, 1\}$ ,  $n_t$  is the mass of agents currently in network 1, i.e.,  $n_t = \int_0^1 a_{i,t} di$ , and  $\nu > 0$  is a parameter measuring the relative importance of strategic complementarities. The fundamental  $\theta_t^j$  follows a Brownian motion with drift  $\mu_j$  and variance  $\sigma_j^2$ .

The relative payoff function, i.e., the difference between the flow utilities in network 1 and in network 0, can be written as

$$\pi(\theta_t, n_t) = \theta_t + \gamma n_t,$$

where  $\theta_t \equiv \theta_t^1 - \theta_t^0 - \nu$  and  $\gamma \equiv 2\nu$ . Notice  $\theta$  follows a Brownian motion with drift  $\mu = \mu_1 - \mu_0$  and variance  $\sigma = \sigma_0^2 + \sigma_1^2$ .

An agent who receives an opportunity to revise her choice at time  $\tau$  will choose action  $a_i = 1$  (that is, will join network 1) whenever the discounted relative payoff of doing so is positive:

$$\mathbb{E} \int_{\tau}^{\infty} e^{-(\rho+\delta)(t-\tau)} (\theta_t + \gamma n_t) dt > 0. \tag{1}$$

If the inequality is reversed, the agent will choose  $a_i = 0$ .<sup>9</sup>

We can apply the results in Frankel and Pauzner (2000) to show that this game will present a unique equilibrium, which is characterized by a threshold in the  $\mathbb{R} \times [0, 1]$  space. For a given network size  $n$ , agents choose 1 if the relative quality of this network ( $\theta$ ) is high enough, and 0 otherwise. We focus on the case of very small shocks to fundamentals ( $\mu, \sigma \rightarrow 0$ ) in order to be able to derive a closed-form expression for the equilibrium threshold, which is presented in Proposition 1.

We now look at this problem from the central planner’s perspective. At every point in time, the planner decides the proportion  $\phi_t$  of agents with an opportunity to revise their actions that will opt for network 1. At time  $\tau$ , the planner maximizes the discounted sum of payoffs across agents, i.e.,

$$\mathbb{E} \int_{t=\tau}^{\infty} e^{-\rho(t-\tau)} [n_t u_t^1 + (1 - n_t) u_t^0] dt,$$

<sup>8</sup> This setting can be also interpreted as an overlapping generations model as in Matsuyama (1991): agents choose a network at birth and are stuck with this choice for life. They face a constant instantaneous probability of death  $\delta$  throughout their lifetime. The birth rate is also  $\delta$ , so total population size is constant. In the QWERTY application, it means that people learn how to type using one kind of keyboard and never switch, but new generations may adopt different standards.

<sup>9</sup> The discounted payoff includes the factor  $e^{-\delta(t-\tau)}$ , which is the probability of not being drawn by the Poisson process from  $\tau$  to  $t$ , i.e., the probability of still being committed to the current choice of network or, in the alternative OLG interpretation, the probability of being alive at  $t$ .

which is equivalent to maximizing

$$\mathbb{E} \int_{t=\tau}^{\infty} e^{-\rho(t-\tau)} \left[ n_t (\theta_t - v) + \gamma n_t^2 \right] dt. \tag{2}$$

Suppose that a proportion  $\phi_\tau \in [0, 1)$  is optimal for the planner at time  $\tau$  and consider the following deviation: the planner chooses  $\phi_\tau = 1$  at time  $\tau$  and future choices for any realization of shocks are kept unchanged. This deviation implies an infinitesimal increase in  $n_\tau$  by  $dn_\tau$ . Its effect on  $n_t$  is given by  $dn_t = dn_\tau e^{-\delta(t-\tau)}$ , since the initial increase in  $n_\tau$  depreciates at a rate  $\delta$ . A necessary condition for optimality is that such deviation is not profitable. Using (2), if a proportion  $\phi_\tau \in [0, 1)$  is optimal for the planner, it cannot be the case that

$$\mathbb{E} \int_{\tau}^{\infty} \frac{\partial \left[ e^{-\rho(t-\tau)} (n_t (\theta_t - v) + \gamma n_t^2) \right]}{\partial n_t} \frac{dn_t}{dn_\tau} dt > 0,$$

which can be written as

$$\mathbb{E} \int_{t=\tau}^{\infty} e^{-(\rho+\delta)(t-\tau)} \left[ \theta_t - v + 2\gamma n_t \right] dt > 0. \tag{3}$$

Hence if the condition in (3) holds, action 1 must be optimal.

The same reasoning implies that action 0 is optimal if

$$\mathbb{E} \int_{t=\tau}^{\infty} e^{-(\rho+\delta)(t-\tau)} \left[ \theta_t - v + 2\gamma n_t \right] dt < 0. \tag{4}$$

The expression for the planner in (4) is very similar to the condition for an agent in (1). The only substantial difference is that the externality is more important for the planner (notice  $\gamma$  is multiplied by 2), as the planner takes into account the spillovers on others.

Mathematically, the planner’s problem is very similar to the agents’ problem in the decentralized equilibrium. At each moment in time, the planner chooses according to (3) and (4), considering that the future path of  $n$  must be consistent with optimality at every point, i.e.,  $\{n_t\}_\tau^\infty$  must result from optimal actions of the planner’s future selves at all dates.<sup>10</sup> Hence this problem is isomorphic to a problem solved by agents with flow-payoffs given by  $\theta_t - v + 2\gamma n_t$ .

Therefore, the planner also chooses according to a downward sloping threshold and the results in Frankel and Pauzner (2000) can be applied here. Proposition 1 presents the planner’s solution and the decentralized equilibrium.

**Proposition 1.** Consider the case of very small shocks,  $\mu, \sigma \rightarrow 0$ .

The decentralized equilibrium prescribes choosing network 1 whenever  $\theta_t > Z^*(n_t)$  and 0 otherwise, where  $Z^*$  is given by

$$Z^*(n) = -\frac{\gamma \delta}{\rho + 2\delta} - \frac{\gamma \rho}{\rho + 2\delta} n. \tag{5}$$

<sup>10</sup> There are no commitment issues in the planner’s problem, since there is no time-inconsistency in preferences and the planner decides on everyone’s actions.

The planner’s solution prescribes choosing network 1 whenever  $\theta_t > Z^P(n_t)$  and 0 otherwise, where  $Z^P$  is given by

$$Z^P(n) = -\frac{\gamma\delta}{\rho + 2\delta} + \frac{\gamma\rho}{2(\rho + 2\delta)} - \frac{2\gamma\rho}{\rho + 2\delta}n. \tag{6}$$

**Proof.** See Appendix A.  $\square$

The weight given by the planner to the current size of the network in (6) is twice as large as the weight given by agents in the decentralized equilibrium in (5). Conventional wisdom might suggest that the planner would push the agents towards the best “fundamentals” (1 when  $\theta$  is high, 0 when  $\theta$  is low) but the planner actually cares less about fundamentals than agents do. If agents prefer the QWERTY over the fundamentally more efficient Dvorak (i.e., if agents prefer to choose 0 even though action 1 would be the optimal choice if  $\theta$  were the only relevant factor), the planner would be even more inclined to choose the fundamentally worst option. Intuitively, the planner takes into account the externality on others that agents fail to internalize, while the intrinsic quality of each good is fully taken into account by agents in the decentralized equilibrium.<sup>11</sup>

The planner would choose the more efficient Dvorak keyboard style if all agents’ machines were to be replaced at a given point in time, while the agents problem in a static setting would exhibit multiple equilibria. However, this result does not apply to a dynamic environment with staggered decisions.

Fig. 1 depicts the results in Proposition 1. To the right of  $Z^*$ , everyone who gets the chance to choose a network chooses 1, so  $n$  goes up, and the opposite happens to left of  $Z^*$ . The planner rotates the threshold so that its slope is half of the slope of the threshold in a decentralized equilibrium, which means  $n$  is relatively more important for the planner.<sup>12</sup>

An important conclusion here is that if the worst network prevails in equilibrium, it is surely efficient, while shifts to the best network might be inefficient. The shaded area is where inefficient shifts to the intrinsically better option happen.<sup>13</sup> Agents start a switch to the smaller network because they take fundamentals into account but do not consider the harm this shift imposes on the large amount of agents still stuck in the (fundamentally) worst option. The planner would prevent such shifts from happening. A larger difference in fundamentals is required to make it optimal to start a shift towards the best (but smallest) network.

### 3. Final remark

We use the QWERTY vs. Dvorak keyboard case as an example of situations where there are strategic complementarities in agents decisions and the prevailing standard is not considered to

<sup>11</sup> The efficiency results here contrast with those in models with information externalities that generate herd behavior (e.g., Bikhchandani et al., 1992). In those models, agents follow others too much from a social point of view. Here, conformity of behavior arises because of preferences, not through learning, and they follow others too little.

<sup>12</sup> Here, network effects are symmetric, meaning that the benefit of a marginal increase in  $n$  for agents in network 1 is the same as the benefit of an increase in  $1 - n$  for agents in network 0. In an alternative setting with asymmetric network effects – with utilities given by  $u_t^0(\theta_t^0, n_t) = \theta_t^0 + v^0(1 - n_t)$ ,  $u_t^1(\theta_t^1, n_t) = \theta_t^1 + v^1 n_t$  and  $v^1 \neq v^0$  – the planner would not only rotate the threshold but also shift it in order to enlarge the region where agents opt for the network that generates larger externalities. An analysis of this case is available upon request.

<sup>13</sup> Notice when  $n = 0.5$ ,  $Z^* = -\gamma/2$ . This is the point at which  $\theta^1 = \theta^0$ . We can think of a vertical line crossing  $(Z^*(0.5), 0.5)$  as a dividing line between the regions where network 0 or network 1 are “intrinsically better”.

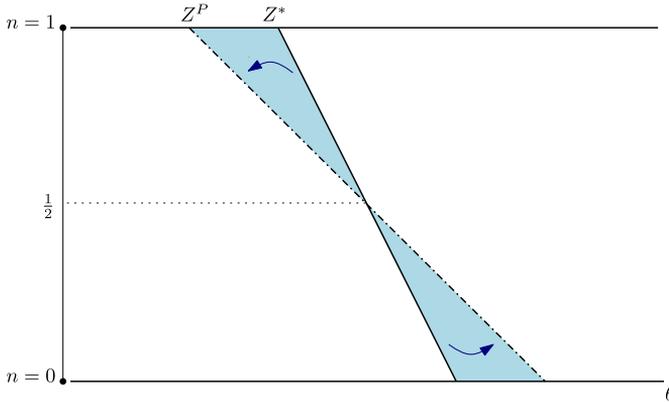


Fig. 1. Planner's solution.

be the best one. When there are two competing standards (or networks), if everyone were to make a choice at the same time (in our example, if all keyboards in the world were to be replaced at once) the efficient solution would be to choose the best one. However, since there are timing frictions and people do not switch from one network to another all at the same time, the efficient solution differs from conventional wisdom: if the worst standard prevails, this is surely efficient, and a central planner would be even more inclined towards the worst option. In fact, there are inefficient shifts to the best network in equilibrium.

**Appendix A. Proof of Proposition 1**

Theorem 1 in Frankel and Pauzner (2000) ensures equilibrium uniqueness in our environment, and Theorem 2 helps us compute the decentralized equilibrium threshold when  $\mu, \sigma \rightarrow 0$ . Applying the latter, we have that the threshold must satisfy the following expression:<sup>14</sup>

$$(1 - n_0) \int_0^\infty e^{-(\rho+\delta)t} \pi(Z^*, n_t^\uparrow) dt + n_0 \int_0^\infty e^{-(\rho+\delta)t} \pi(Z^*, n_t^\downarrow) dt = 0, \tag{7}$$

where  $n_t^\uparrow = 1 - (1 - n_0)e^{-\delta t}$  and  $n_t^\downarrow = n_0e^{-\delta t}$ . Solving the equation above for  $Z^*$  gives us the agents' threshold in a decentralized equilibrium.

The remaining of the proof follows from the discussion in the main text. We have shown that solving the planner's problem is equivalent to solving the game agents play, but with flow-payoffs given by  $\theta_t - \nu + 2\gamma n_t$ . Thus the planner's solution is the threshold  $Z^P(n_0)$  that solves

<sup>14</sup> Burdzy et al. (1998) show that, when  $\mu, \sigma \rightarrow 0$ , at any point  $(Z^*(n_0), n_0)$  along the threshold, it takes no time for  $n$  to start moving in one direction, and that it never comes back, i.e.,  $n$  either grows continuously towards 1 or decreases towards 0. They also show that the probability of  $n$  bifurcating up is exactly  $1 - n_0$ , and the probability of  $n$  bifurcating down is  $n_0$ . These probabilities can be used to pin down an agent's beliefs along the threshold. An agent at  $(Z^*(n_0), n_0)$  assigns probability  $1 - n_0$  to an upward bifurcation followed by the increase of  $n_t$  at rate  $\delta(1 - n_t)$  – since to the right of the threshold everyone who gets the chance to revise actions is going to choose action 1 – and probability  $1 - n_0$  to a downward bifurcation followed by the decrease of  $n_t$  at rate  $-\delta n_t$ . Equation (7) states that an agent with such beliefs must be indifferent between actions 0 and 1 at any point along the equilibrium threshold.

$$(1 - n_0) \int_0^{\infty} e^{-(\rho+\delta)t} \left( Z^P - v + 2\gamma n_t^{\uparrow} \right) dt$$

$$+ n_0 \int_0^{\infty} e^{-(\rho+\delta)t} \left( Z^P - v + 2\gamma n_t^{\downarrow} \right) dt = 0. \quad \square$$

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